

## Problem 2.18

[Difficulty: 2]

**2.18** Air flows downward toward an infinitely wide horizontal flat plate. The velocity field is given by  $\vec{V} = (ax\hat{i} - ay\hat{j})(2 + \cos \omega t)$ , where  $a = 5 \text{ s}^{-1}$ ,  $\omega = 2\pi \text{ s}^{-1}$ ,  $x$  and  $y$  (measured in meters) are horizontal and vertically upward, respectively, and  $t$  is in s. Obtain an algebraic equation for a streamline at  $t=0$ . Plot the streamline that passes through point  $(x, y) = (3, 3)$  at this instant. Will the streamline change with time? Explain briefly. Show the velocity vector on your plot at the same point and time. Is the velocity vector tangent to the streamline? Explain.

**Given:** Time-varying velocity field

**Find:** Streamlines at  $t = 0$  s; Streamline through (3,3); velocity vector; will streamlines change with time

**Solution:**

For streamlines 
$$\frac{v}{u} = \frac{dy}{dx} = -\frac{a \cdot y \cdot (2 + \cos(\omega \cdot t))}{a \cdot x \cdot (2 + \cos(\omega \cdot t))} = -\frac{y}{x}$$

At  $t = 0$  (actually all times!) 
$$\frac{dy}{dx} = -\frac{y}{x}$$

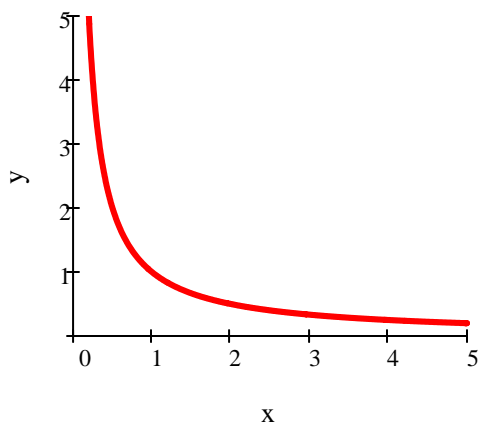
So, separating variables 
$$\frac{dy}{y} = -\frac{dx}{x}$$

Integrating 
$$\ln(y) = -\ln(x) + c$$

The solution is 
$$y = \frac{C}{x}$$
 which is the equation of a hyperbola.

For the streamline through point (3,3) 
$$C = \frac{3}{3} \quad C = 1 \quad \text{and} \quad y = \frac{1}{x}$$

The streamlines will not change with time since  $dy/dx$  does not change with time.



At  $t = 0$  
$$u = a \cdot x \cdot (2 + \cos(\omega \cdot t)) = 5 \cdot \frac{1}{s} \times 3 \cdot \text{m} \times 3$$

$$u = 45 \cdot \frac{\text{m}}{\text{s}}$$

$$v = -a \cdot y \cdot (2 + \cos(\omega \cdot t)) = 5 \cdot \frac{1}{s} \times 3 \cdot \text{m} \times 3$$

$$v = -45 \cdot \frac{\text{m}}{\text{s}}$$

The velocity vector is tangent to the curve;

Tangent of curve at (3,3) is 
$$\frac{dy}{dx} = -\frac{y}{x} = -1$$

Direction of velocity at (3,3) is 
$$\frac{v}{u} = -1$$

This curve can be plotted in Excel.